A Bayesian algorithm for real-time model selection in caustic-crossing events

Noé Kains (ESO)
Keith Horne, Paul Browne, Markus Hundertmark (St Andrews)
Arnaud Cassan (IAP)
How things work

• Follow-up teams select “promising” MOA/OGLE events, higher cadence or/and better photometry (with e.g. lucky imaging)

• Challenge: new-generation surveys alert ~1700 events/ year
  • How to select best events to follow up?
  • How to model so many events, given the large amount of (human and computational) time required for each event
Microlensing Anomalies

• Following up the “right” anomalies is key
• No simple selection criterion
• Come in a huge variety of shapes and timescales (few hours - few days)
• No way of avoiding computationally intensive modelling
What do we want from modelling

• 2 main purposes:
  • Real-time modelling → predictions of upcoming features, feed information back to telescopes for optimal target selection
  • Post-event modelling: be as thorough and systematic as possible (i.e. explore parameter space)

• Ideally, an algorithm that is good for both
• Several approaches amongst modellers (see talks by V. Bozza, C. Han)
Feature-based parameterisation

• Use parameters that are well constrained by data features

• Caustic-crossing events:
  • Re-parameterised binary-lens lightcurves (Cassan+ 2008, 2010 Kains+ 2009) + MCMC to conduct grid search and generate posterior maps with available data
  • Include as much prior information as possible without biasing modelling
Chapter 4. The basis for an automatic binary-lens fitting algorithm

Figure 4.1: The alternative binary-lens parameterisation used in this discussion.

Top: The \( s \) coordinate, which runs along the caustic folds from \( s = 0 \) to \( s = 2 \).

Bottom: The source enters the caustic at time \( t = t_{\text{in}} \); its location on the caustic fold at that time is \( s(s_{\text{in}}) \). Similarly, the source exits the caustic at \( t = t_{\text{out}} \), at which point it is located at \( s(s_{\text{out}}) \). The example caustic used here is for parameters \( d = 1.5, q = 0.5 \).

This parameterisation is illustrated on Fig. 4.1 and Fig. 4.2. These four parameters (in addition to \( d, q \) and \( \rho^* \)) which describe the caustic crossings therefore also define an alternative parameterisation of the binary lens, far better suited to describing the problem at hand.

4.2.2 Exploration of the parameter space

We start by exploring a wide region of the parameter space with a \((d, q)\) grid regularly sampled on a logarithmic scale. This choice comes from the fact that the size of the caustic structure depends on the parameters.

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Parameterisation of Cassan (2008)

Locations on the caustic of caustic entry and exit

\[ s_{\text{in}}, s_{\text{out}} \]

Times of caustic entry and exit

\[ t_{\text{in}}, t_{\text{out}} \]

Duration of crossing \( \Delta t_c \)

\[ t_0, t_E, \alpha, u_0, \rho^*(or \ t_\ast), d, q \]

\[ \rightarrow t_{\text{in}}, t_{\text{out}}, s_{\text{in}}, s_{\text{out}}, \Delta t_c, d, q \]
Assuming uniform distribution of source trajectories and source sizes, this is what a map of a joint prior on (sin, sout) looks like.

We can exploit the structure of these maps to devise a scheme for thorough parameter space exploration.
Each ‘sub-box’ corresponds to the source entering and exiting the caustic on a different caustic fold.

By exploring each sub-box, we can ensure all possible caustic-crossing trajectories are explored for a given caustic.

This allows us to conduct a grid search in the (d, q) plane.
1. (d, q) grid
2. Select caustic
3. Explore P(s_{in}, s_{out}) sub-boxes for each caustic
4. Find best fit(s) for each (d, q); iterate
...end up with a “posterior map” of whichever badness-of-fit statistic we choose. E.g. chi2 map:

Separation between lens components (e.g. planetary orbit)

OB070472, first analysed with no priors in Kains+ 2009, also Kains+ in preparation
A robust statistic

- Kains+ (2009): best-$\chi^2$ model for OB07472 has extreme $t_E (~4000$ days)
- $\chi^2$ not necessarily the best estimator of true parameters
  - Need for a more reliable badness-of-fit statistic?
  - Include priors
- Some alternatives:
  - Maximum a-posteriori (MAP): Maximise $\chi^2 - 2*\ln(\text{prior})$
  - MAP + volume: take into account parameter space volume
  - Bayesian Information Criterion (BIC): takes into account number of parameters and data points
Choosing priors

• Free to choose prior from any suitable source
• E.g. a prior of timescales could come from a distribution of past observed events or a model distribution (e.g. Wood & Mao 2005)
• We used a joint prior on tE and \( \rho_* \) obtained from a Besançon model simulation (Robin+ 2003)
BIC = \(X^2 - 2 \ln(\text{Prior}) + N_{\text{eff}} \cdot \ln(N_D)\)
Conclusions

• Neat way of ensuring the parameter space is explored completely
• Systematic and (nearly) automatic analysis of events
• Improved badness-of-fit statistic allows us to locate more robust best-fit minimum
• Drawbacks: limited to caustic-crossing events for now
• Could extend the parametrisation to include non-crossing caustic approaches etc.