Difference Imaging in Wide Field Surveys

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What is difference imaging and why do we want to use it?

How to implement a spatially varying numerical kernel

How to implement difference imaging on a GPU

Large Images
Outline

1. What is difference imaging and why do we want to use it?
2. How to implement a spatially varying numerical kernel
3. How to implement difference imaging on a GPU
4. Large Images
What is difference imaging and why do we want to use it?

What is difference imaging?

- method to detect transient events (including microlensing)
- method to get quality time series photometry of transient events (and variable stars)
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MOA implementation

- MOA camera: $10 \times 2048 \times 4096$ CCD mosaic
- Exposure time: 60 secs (for Bulge fields) – generate difference images for all frames in an exposure – get positions of possible transients – generate “quicklook” photometry for all transients in the list
- All done in real time, i.e before the next exposure finishes
- This approached has worked well for MOA
Can we do the same for new generation surveys

- KMT
  - Camera: $4 \times 9K \times 9K$

- LSST
  - Camera: $189 \times 4K \times 4K$

Important issues

- Can difference imaging be done fast enough for real-time?
- Can it work on large images
Principles of Difference Imaging

- Given two images, \( r \) and \( i \), of the same field taken at different times, one constructs a difference image by
  \[
d(x, y) = i(x, y) - (r * k)(x, y) - b(x, y)
\]
- Solve for the “convolution kernel”, \( k \), and the differential background, \( b \)
- Both \( k \) and \( b \) are spatially varying
  – major issue for large images!
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Classic Method

- Model kernel as a linear combination of smooth analytic basis functions (Alard & Lupton 1998)
- Model spatial variation of kernel by 2D polynomial (applied to each coefficient)
- Model background as 2D polynomial

Numerical trick by Alard (1999)

- Kernel is constant on over its own size
- Select a number of “stamps” across the image
- Build zero order matrices for each stamp
- Expand into linear system for the whole image by combining zero order linear systems for each stamp with the basis functions for the spatial variations
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Stamps
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**Classic Method**

- Size of the linear system:
  \[ N = 1 + (N_k - 1)N_s + N_b \]

- Typically
  \[ N_k = 49 \text{ (kernel basis functions)} \]
  \[ N_s = 6 \text{ (functions to model spatial variations in } k \text{)} \]
  \[ N_b = 6 \text{ (functions to model background)} \]
  \[ N = 295 \]

- All takes \(<10 \text{ seconds on a modern PC for a single 2K } \times \text{ } 4K \text{ image} \]
- Fast enough? Maybe.
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**Numerical Kernel**

- Treat kernel as a numerical table and solve for each entry (Bramich 2008)
- Used extensively for follow-up observations (small telescopes, small CCDs)
  - weird kernel shapes, not necessary to interpolate (Albrow 2009)
- More computationally expensive than the classic method
- Can we use the numerical kernel for wide field surveys?
  - Need to allow for spatially varying kernel
  - Need a reasonable computation time
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A spatially varying numerical kernel can be implemented using the principles of the classic method.

The basis function set is just a grid of delta functions.

Modify slightly to apply flux conservation rule:

\[ k(x, y) = c_0 \delta(x, y) + \sum_n c_i [\delta(x - j, y - i) - \delta(x, y)] \]

The \( c_i \) are spatially dependent (modelled using 2d polynomial set), but the photometric scaling is always given by \( c_0 \).

Also include background

\[ b(x, y) = \sum_n c_n x^i y^j \]
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Spatially varying numerical kernel

A numerical 5×5 kernel basis set
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Linear system becomes large

- For a $17 \times 17$ numerical kernel, we need to build a $289 \times 289$ zero order matrix for each stamp
- Degree 2 polynomial for spatial variations in the kernel (6 basis functions)
- Degree 2 polynomial for background (6 basis functions)
- Total size of linear system
  \[ N = 1 + 288 \times 6 + 6 = 1735 \]

Can we speed things up?
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NVIDIA GeForce GTX 480
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Programming a GPU

GPU architecture supports **Single Instruction Multiple Thread** programming model

- executes instruction for all threads in the warp, before moving onto next instruction
- divergence occurs when threads in a **warp** follow different control flows. Sequential passes are then needed which can affect performance
- but **SIMT** approach is ideal for most image processing operations
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Examples

- **Zero order matrices**

  \[ Q_{sij} = \sum_{(x,y) \in S_s} [r(x - x_i, y - y_i)r(x - x_j, y - y_j) - r(x, y)r(x, y)] \]

  each thread works on its own \( s, i, j \)

- **Expanding zero order matrices**

  \[ M_{ij} = \sum_{\text{stamps}} F(s, s_i)F(s, s_j)Q(s, k_i, k_j) \]

  one thread per \( i \) and \( j \)

- **Convolution**

  \[ (r \ast k)(x, y) = \sum_{(x_k, y_k)} r(x_k, y_k)k(t, x_i, y_i) \]

  one thread per pixel \( x, y \) (\( t \) is “tile” index)
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Performance

<table>
<thead>
<tr>
<th>Operation</th>
<th>CPU</th>
<th>GPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Build zero order system</td>
<td>30 secs</td>
<td>2 secs</td>
</tr>
<tr>
<td>Expand over stamps</td>
<td>6 secs</td>
<td>72 ms</td>
</tr>
<tr>
<td>Solve linear system</td>
<td>10 secs</td>
<td>10 secs</td>
</tr>
<tr>
<td>Build difference image</td>
<td>5 secs</td>
<td>91 ms</td>
</tr>
</tbody>
</table>

2 K × 4 K image
17 × 17 spatially variable numerical kernel
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Dealing with large images

Work in progress

- For large images, 2nd order polynomials may not be able to model high frequency spatial variations in the kernel and background.
- One approach is to divide image into tiles and solve separately for each.
- May have issues with continuity across the tile boundaries.
- Can we force continuity conditions on the polynomial models?
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Hermite polynomials

Form a set of 16 2D polynomials, $h_{ij}(u, v) = h_i(u)h_j(v)$ from the 1D Hermite blending functions

$h_1(u) = 1 - 3u^2 + 2u^3$
$h_2(u) = u - 2u^2 + u^3$
$h_3(u) = 3u^2 - 2u^3$
$h_4(u) = -u^2 + u^3$

where $u = (x - x_i)/w$, $v = (y - y_i)/w$ for tile $i$

- each tile has its own 16 2D polynomial set
- but coefficients are shared at each node of the tile
Hermite polynomials

- for any arbitrary set of coefficients, the resulting surface satisfies at the tile boundaries:
  - $C_0$: functions continuous
  - $C_1$: 1st derivatives continuous
- Total of $4(N_x + 1)(N_y + 1)$ coefficients
- Could use this basis set to model spatial variations in the kernel, and background
- End up solving a single large linear system
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Tiling