Space-Based Imaging Astrometry: Life with an Undersampled PSF

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Overview of the Talk

• Astrometry with HST
  – 3 critical issues
  – Science
    • General
    • Microlensing

• Extensions to WFIRST
Astrometry with HST

• One of the original selling points
  – FGS: always planned
  – Also intended imaging astrometry

• Several challenges
  1) Undersampling → PSFs
  2) Distortion (several sources)
  3) Differential astrometry → Transformations
     .... took several years to address these issues.

Goal of talk: an appreciation of the issues and possibilities
Astrometry: Fundamental limitations

- **Poisson statistics**
  - Gaussian PSF
    \[ \delta x \sim \sigma_x / \sqrt{N} \]
  - Best position: straight centroid

- **Pixelization**
  - Complication: loses information
  - Simple centroid no longer works
  - Requires good PSF
Illustration of Undersampling

Where is the center?

We need an accurate PSF!
Undersampling and Astrometry

• Impossible?
  – A point source has “no hair”
  – Overconstrained problem
    • 3 parameters (x,y,f), ~9 pixels
  – Minimal requirements: “slosh”
    • Only pathological if FWHM < 1 pixel

• What is possible?
  – 0.005-0.01 pixel possible ~ (S/N)^{-1}
  – Need good PSF model
  – Need good dithering

• Limitations
  – Individual images; do not use stacks
  – Harder in crowded/sparse fields
  – Ideal in “semi-crowded” regime
PSFs: Photometry -vs- Astrometry

- Photometry: how much flux is there? (SUMS)
- Astrometry: where is the flux? (DIFFERENCES)
- Shape: exactly where is flux... (DIFFs of DIFFs)
  - All require good PSF, but they make different demands

PSF Modeling
- Ground
  - Variable-seeing dominated
  - Gaussian-fitting models, DAOPhot
- HST
  - Stable but undersampled, new regime
  - Exquisitely precise models possible
What do we mean by the PSF?

• $\psi_{\text{INST}}(\Delta x, \Delta y)$: the “Instrumental” PSF:
  – The PSF as it hits the detector
  – Good theoretical motivations: Gaussians, Moffat
  – See $\psi_{\text{INST}}$ only indirectly in images
    • Must deconvolve the PSF from the pixels
    • Saving grace: often solve for limited set of parameters

• $\psi_{\text{EFF}}(\Delta x, \Delta y)$: the “Effective” PSF:
  – The PSF after pixelization: $\psi_{\text{EFF}} = \psi_{\text{INST}} \otimes \Pi$
  – Empirical: no natural basis function to describe
  – Tod Lauer’s 1999 tutorial in PASP on image reconstruction
    • OLD: Pixels as light buckets
    • NEW: Pixels as point-samplings of a continuous scene
  – Epiphany: we never deal with anything BUT the effective PSF
    • See $\psi_{\text{EFF}}$ directly in images
    • Can measure $\psi_{\text{EFF}}$ directly from images
The “Effective” PSF

• What it represents:
  – Fraction of light that falls in a pixel, relative to the center of the star

• Modeling images:

OLD: \( P_{ij} = S + F_* \times \int \int_{x,y \in (i,j)} \psi_{\text{INST}}(x-x_*, y-y_*) \, dx \, dy \)

NEW: \( P_{ij} = S + F_* \times \psi_{\text{EFF}}(i-x_*, j-y_*) \)

• How to “see” it:

\[ \psi_{\text{EFF}}(\Delta x, \Delta y) = (P_{ij} - S)/F_* \]

  – Where: \( \Delta x = i - x_* \), etc
  – We have to know \((x_*, y_*)\) and \(F_*\)
How a single star samples $\psi_E(\Delta x, \Delta y)$

- A single star has an array of pixels about its center.
- Each pixel contains a fraction of its flux.
- Each pixel reports $\psi_E$ at one point in $\psi_E$’s domain.
How two stars sample \( \psi_E (\Delta x, \Delta y) \)

- In general, the two stars will be at different pixel phases.
- This gives us a different array of samples of \( \psi_E \)
How three stars sample $\psi_E(\Delta x, \Delta y)$

- A third star will give yet more variety in our sampling of $\psi_E$
How 200 stars sample $\psi_E(\Delta s, \Delta y)$

- A large number of stars gives us an almost even coverage of $\psi_E$ across its 2-D domain.
How to solve for $\psi_E(\Delta x, \Delta y)$

- A regularly-spaced array of grid-points
- Specify value of $\psi_E$ at those points to best-fit the data.
“Seeing” $\psi_{\text{EFF}}$ Directly
The model of $\psi_E(\Delta x, \Delta y)$

- Tabulated values of $\psi_E$ at this array of points across its domain.
How to use $\psi_E(\Delta x, \Delta y)$

Need to know:
“What fraction of light should land in a pixel, if the pixel is centered at $(\Delta x, \Delta y)$ relative to the point source?”

Need to interpolate:
→ Use bi-cubic interpolation
1) How to find the PSF?
2) How to use the PSF?
1) How to find the PSF?

2) How to use the PSF?

Fitting for Flux and position:

\[ P_{ij} = S + F_\star \times \psi_{ij} \]

- Nice, linear equation!
- Which pixels to use?

\[ \psi_{ij}(i-x_\star, j-y_\star) \]

\[ F_\star = \text{slope} \]

\[ 20\% \]

assumed center
1) How to find the PSF?

2) How to use the PSF?

Fitting for Flux and position:

\[ P_{ij} = S + F_* \times \psi_{ij} \]

- Nice, linear equation!
- Which pixels to use?
PSF: Finding -vs- Using

- **Degeneracy:**
  - Finding $\psi_{\text{EFF}}$ requires $(x,y,f)$
  - Finding $(x,y,f)$ requires $\psi_{\text{EFF}}$
- **Iteration**
  - Dithers break the degeneracy!
Higher-Level PSF Issues...

- Spatial variability...
Higher-Level PSF Issues...

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Array of PSFs for F606W ACS
Higher-Level PSF Issues...

- Spatial variability...

Core intensity varies by ±10% over scales of ~500 pixels.
Higher-Level PSF Issues...

- Spatial variability
- Time variability
  - Breathing: +/- 2%
  - Hybrid models:
    - $\text{PSF}(x,y;t)=\text{PSF}(x,y)+\text{PSF}(t)$
    - Good for ACS, ok for UVIS
  - Long-term variability (ACS)
- How to define “center”? 
  - Peak? Centroid? Point of Symmetry?
  - Cross-talk with distortion
- Pixel-response function: $\Pi(\Delta x,\Delta y)$
  - Included naturally
- Color variability: $\sim$0.002 pixel (extreme: 0.02 pixel)
ISSUE#2: Distortion

observed position (+5.00, 5.00)
true position
(+5.00, +5.45)

observed position
(+5.00, +5.00)
Sources of Distortion

1) Geometric optics:
   • Linear “skew”: 500 pixels over 2000 → Parallelogram pixels
   • Non-linear: 50 pixels over 2000

2) Filters introduce distortion
   • Offsets, scale changes
   • “Fingerprint” of ~0.05 pixel

3) Detector “stitching” defects
   • WFPC2: every 34.1333\(^{th}\) row 3% shorter
   • ACS/WFC: pattern every 68.2666th column
   • WFC3/UVIS: 2-D zones

4) CTE losses...
   • ACS Solution now available (UVIS coming soon)

Need empirical approach…
Plot everything against everything else…
ISSUE#1: Undersampling/PSFs
ISSUE#2: Distortion
ISSUE#3...

Transformations

All HST astrometry is differential astrometry

→ Guide-star precision ~ 0.5″ (improved from 1.5″!)
→ No reference stars in typical field
→ We never know the true pointing

Always need to define a local reference frame

→ Pixels/positions have only relative meaning.
→ Choosing a frame/population you know something about
  → absolute $\mu = 0$ (galaxies)
  → average $\mu$ = same (clusters)
  → average $\mu$ = unchanging (field)
→ Allow for breathing effects
→ 6-param transformations, or go local
ISSUE#1: Undersampling/PSFs
ISSUE#2: Distortion
ISSUE#3: Transformations

Good News: All manageable issues

Undersampling/PSFs:
→ Ways to model accurately, get 0.01-pixel positions
→ Libraries available, usually sufficient

Distortion:
→ Stable, model available, small variations, ~ 0.01 pixel

Transformations:
→ Can optimize for program

Bad news:

No one-size-fits-all solutions…
Astrometric Science with HST…

1) Cluster Membership
2) Absolute motions
3) Internal motions in clusters
4) Microlensing applications
1) Bulk motions: NGC6397 PI-Rich, UCLA

Proper-Motion Cleaning
4) Microlensing Applications (breaking degeneracies)

1) Color-dependent centroid shift (1\textsuperscript{st} moment)
   - Color difference between lens/source $\rightarrow \mu$

2) De-blending (measure 2\textsuperscript{nd} moment)

3) Astrometry during the event

During event

After event

$f \times \mu = 0.6 \text{ mas/2yr}$

Bennett et al 2006
OGLE-2003/BLG-235
MOA-2003/BLG-53
2) De-blending:

OGLE-2005-BLG-169
Epoch 1 HST Observations
OGLE-2005-BLG-169L
-Wiki: 2000 kpc “bulge” star
-Uranus-mass extrasolar planet

HST IMAGES
- GO-12541 (PI-Bennett)
- 2 orbits Oct 2011
- 6xB, 8xV, 7xI
- decently dithered
Seven F814W Observations
Seven F814W Observations
Seven F814W Observations

$i_3$
Seven F814W Observations

\( i_4 \)
Seven F814W Observations
Seven F814W Observations

\[ i_6 \]
Seven F814W Observations

\[i_7\]
Stacked F814W Observations
Zoomed F814W Stack

Source looks elongated relative to neighbors
Achieved sub-pixel sampling
Stacked F814W Observations
Subtracted F814W Stack

Residuals in X when we subtract a PSF from each image and stack...
Subtracted Neighbor...

PSF IS GOOD!

Almost *no* residuals
When we Subtract a PSF from a (brighter) neighbor
This means that the residuals of the target-star subtraction are *real*. 
2-Source Subtracted F814W

We get very good subtraction residuals when we fit for two sources.
Two-source solution:

• Offset consistent in the B, V and I data:
  – $\Delta x = 1.25$ pixels = 50 mas ($\Delta T = 6$ yrs)
  – $\Delta y = 0.25$ pixel = 10 mas
  – FLUX: (left) (right)
    • F814W 3392 e$^-$ 3276 e$^-$
    • F555W 2158 e$^-$ 3985 e$^-$
    • F438W 338 e$^-$ 1029 e$^-$
3) Astrometry during the event

- Kailash Sahu (PI)
  - Some long-duration event follow ups with HST
    - OGLE-2007-BLG-224
    - MOA-2009-BLG-260
    - MOA-2010-BLG-235
    - MOA-2010-BLG-356
    - MOA-2010-BLG-482
  - GO-12586: Finding our own events
    - 192 orbits over 3 cycles
Fast BH, NS, WD or slow BD?

Schematic of event

Duration of event $\propto$ mass $\times \mu$

Astrometric offset $\propto$ mass

Photometry/Astrometry
OBSERVING STRATEGY

• NUMBER OF TARGETS
  – Each ACS field has ~200,000 stars
    • 50% have S/N > 100
  – Each WFC3/UVIS field has 150,000 stars
  – Total of > 1,500,000 stars

• OBSERVING CADENCE
  – Optimized for long-duration events
  – One visit every 2 weeks over two 4-month windows
    • 64 visits per year

• EXPECTATIONS: (54 / 120 events “astrometric”)
  – 18 events due to BHs
  – 14 due to NSs
  – 22 due to MS stars…. STARTS IN APRIL!
APPLICATIONS TO WFIRST

• HST programs: hard to get time!
  – WFIRST will do for all sources
• Success with HST PSF encouraging
  – Model static part
  – Perturb with time-variable part
  – Need “semi-crowded” star field
    • Construct basis functions for PSF / GC
    • Long stare
    • WL will ♥ μL!
  – Demo software (Sahu program)