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Planetesimal Formation and the Gravitational Instability in Cooling Accretion Discs

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Abstract

Both rocky planets and the cores of gas/ice planets are thought to form through collisional agglomeration, on scales from micron-sized dust grains to kilometre-sized planetesimals (Safronov 1972, Wetherill 1990). The gas drag acting on intermediate metre-sized objects would however cause them to accrete rapidly onto the proto-star, preventing further collisional growth (Weidenschilling 1977). In self-gravitating discs, the presence of spiral arms can act to concentrate solid particles within the arms (Rice et al 2004,2006), boosting planetesimal formation through both enhanced particle densities and increased collisions, both of which are linked to the gas density perturbations of the spiral arms themselves (Rice et al 2004, Haghighipour & Boss, 2003). We show here through controlled numerical experiments that the amplitude of such perturbations critically depends on the cooling rate in the disc and we provide a simple fitting formula for such dependence. This is useful to identify the thermal regimes which are more favourable for planet formation to occur.

Numerical Modelling

Our 3D global simulations of gaseous discs were performed using an SPH code (Monaghan 1992) capable of modelling the disc's self gravity. The discs, which have a mass of 0.1 times that of the proto-star, have a surface density profile $\Sigma \sim R^{-3/2}$ and are initially gravitationally stable throughout. All simulations are conducted in dimensionless units, and each disc contains 500,000 gas particles with radial range $0.25 \leq R \leq 25.0$ orbiting a sink particle, allowing for accretion onto the central proto-star. We use a simple cooling prescription where

$$\Omega t_{cool} = \beta = const$$

throughout an individual simulation. Since Gammie (2001) has shown that for $\beta \leq 3$ fragmentation into bound objects occurs, we use $4 \leq \beta \leq 10$ (cf. Lodato & Rice 2004). The discs therefore settle into a quasi-steady state characterised by the presence of spiral density waves, but do not fragment. The simulations are run for 10 outer cooling times to ensure that they are fully in dynamic thermal equilibrium.

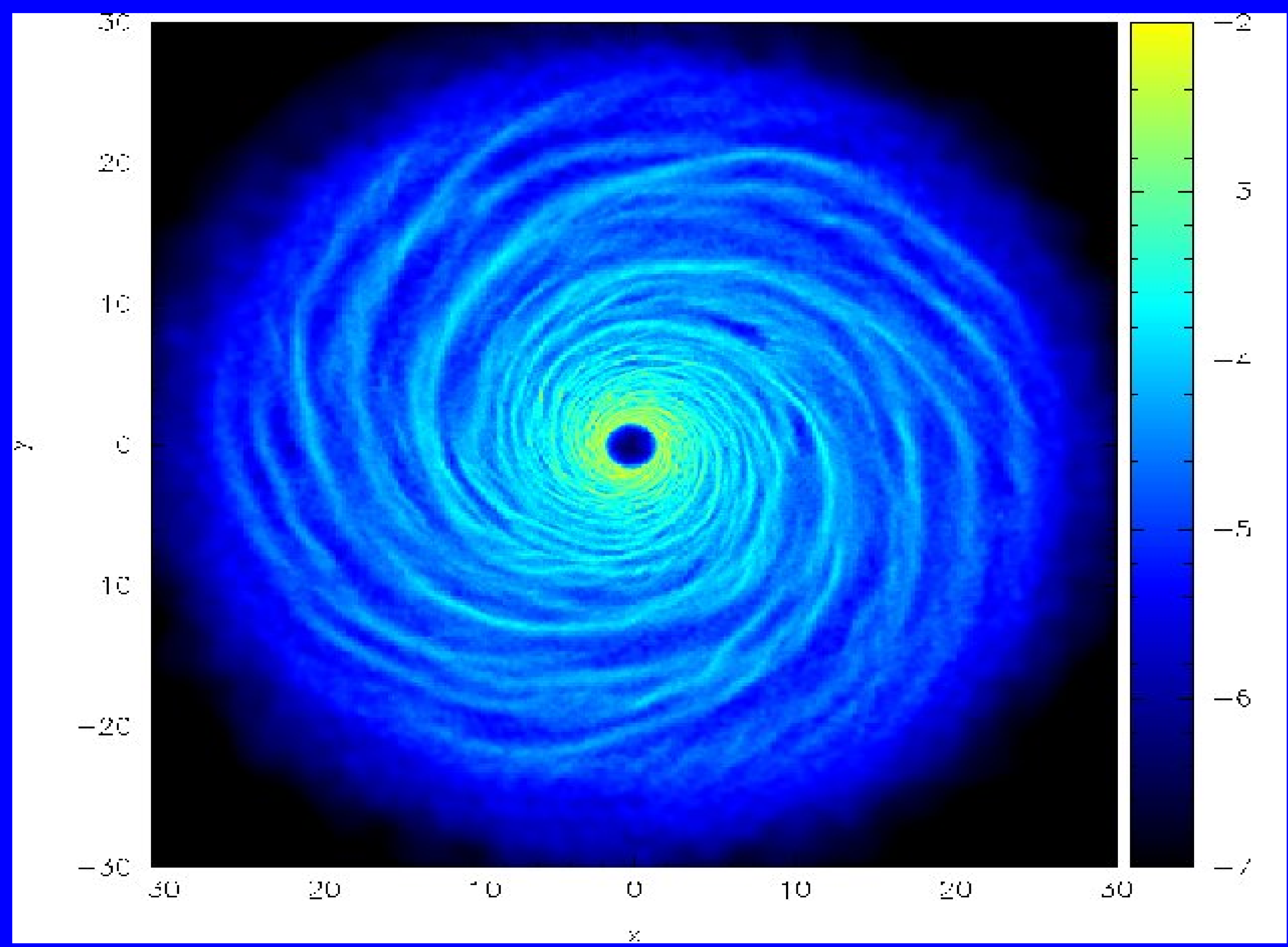

Figure 1: Logarithmic surface density plot where β = 5 generated using SPLASH (Price,2007).

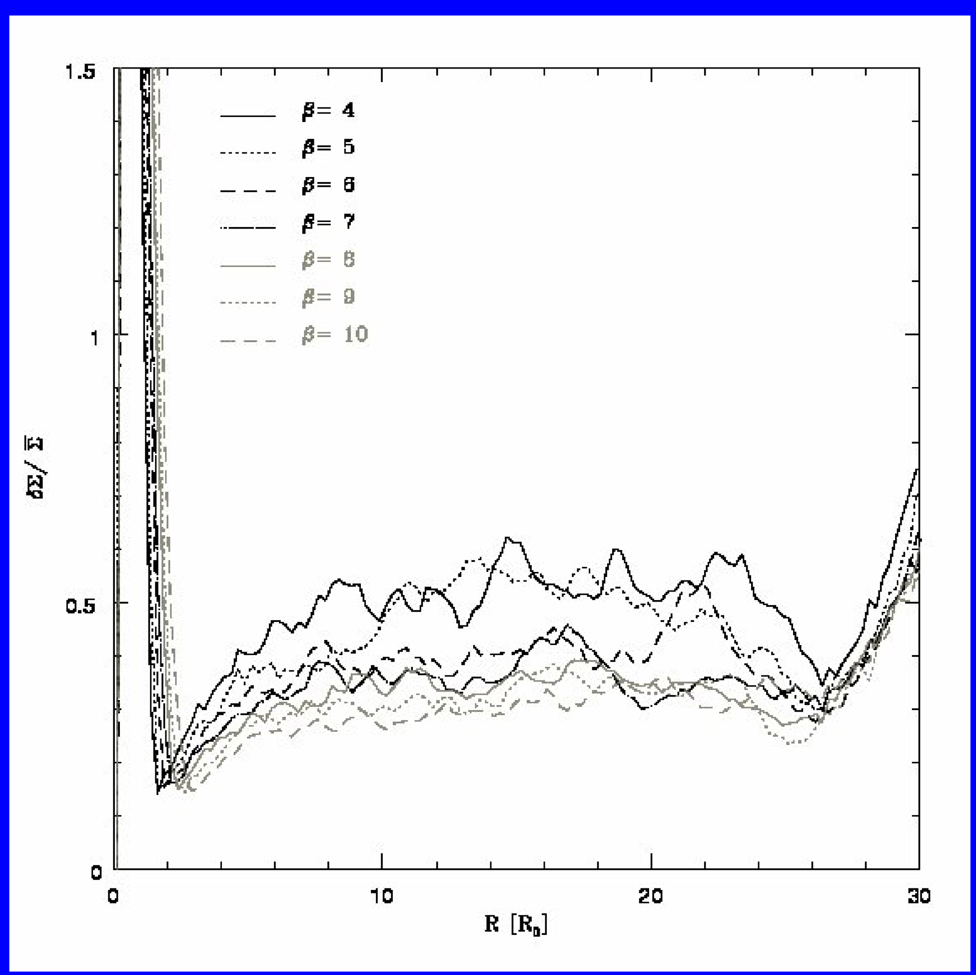

Figure 2: Surface density amplitude variation as a function of radius and β

Results

The discs all rapidly settle into a dynamic equilibrium state where the imposed cooling is balanced by the heating generated through the gravitational instability and where the stability parameter Q is kept close to unity throughout the disc. This quasi-stable self-gravitating state is characterised by the presence of multiple spiral density waves, as shown in Fig. 1. The azimuthally averaged surface density perturbation is shown in Fig. 2. It is clear that the more rapid the cooling the greater the amplitude. This is further shown in Fig. 3, where we have also taken a radial average of the amplitude and plotted it as a function of the cooling rate.

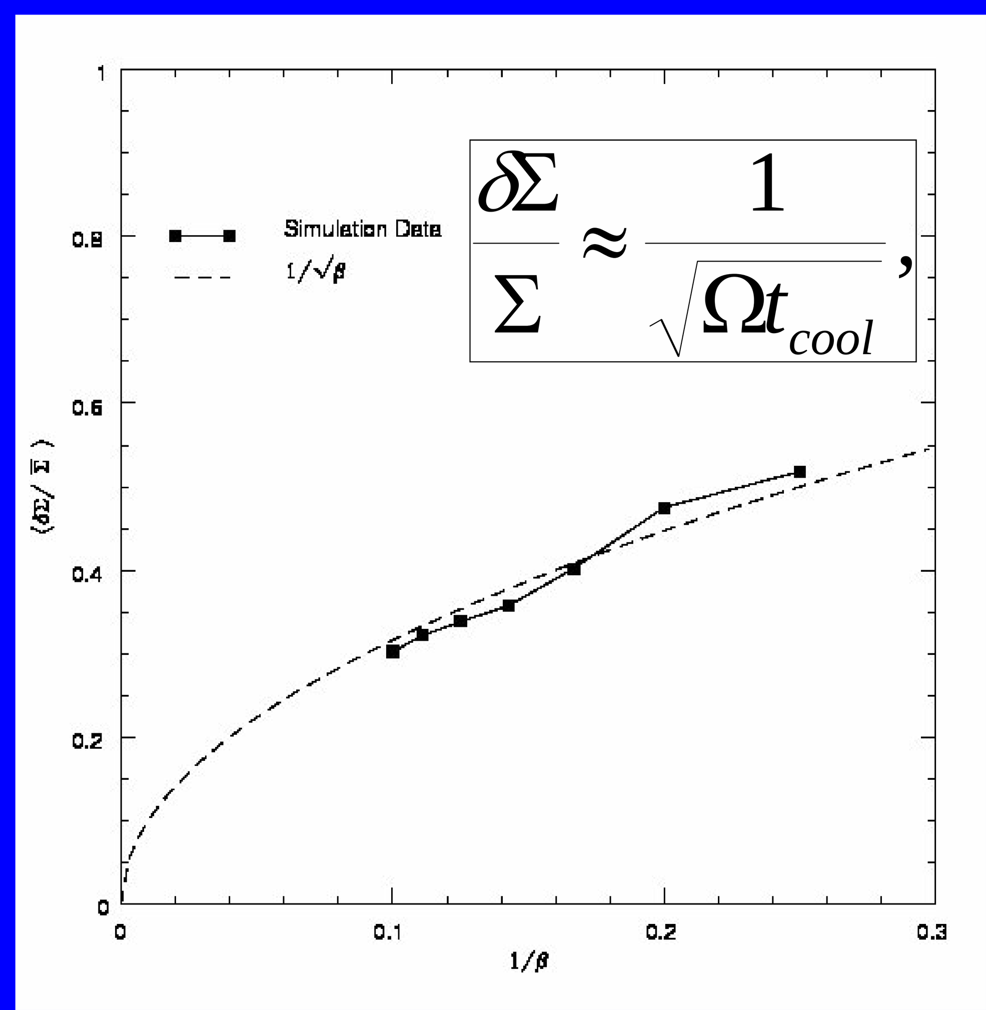

Figure 3: Average surface density perturbation am-

Fourier Analysis

In order to calculate the relevant Mach numbers the radial and azimuthal wavenumbers need to be calculated. Using Fourier analysis of the disc structure we can find the dominant modes at each radius, and these are shown in Fig. 4. For both the azimuthal and radial wavenumbers the amplitude decreases with decreasing cooling strength. In both cases the mode spectra are almost identical, varying only in amplitude. Note that the peak radial wavenumber is approximately equal to the inverse of the local disc scale-height H, such that $kH \approx 1$, as predicted from linear analysis.

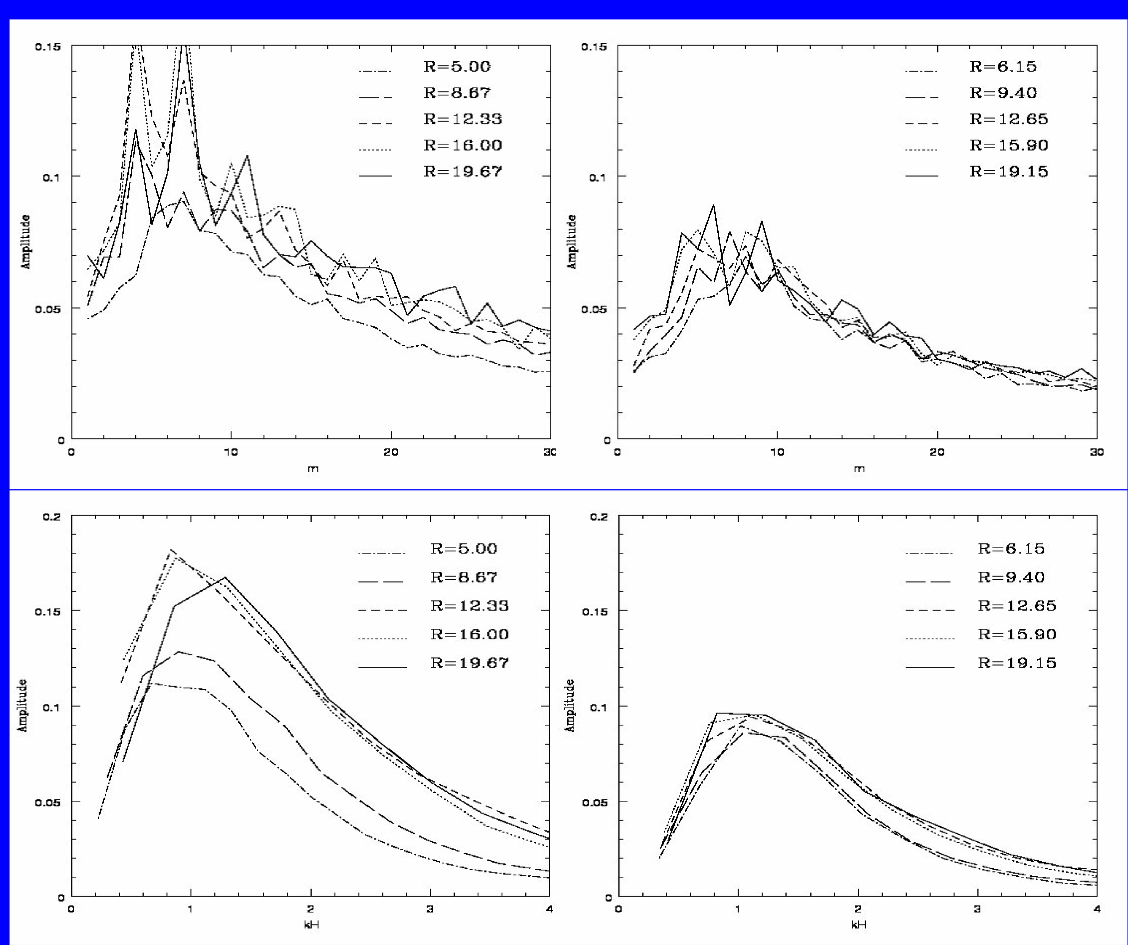

Figure 4: Amplitude of the radial (bottom) and azimuthal (top) wavenumber as a function of radius for β = 5 (left) and β = 10 (right).

Constancy of Mach numbers

By averaging these spectra the relevant Mach numbers are obtained, and these are plotted in Fig. 5. It appears that the self-regulated structure of the disc is such that the Doppler shifted Mach number is exactly unity, implying that the flow into the spiral arms is exactly sonic. Both are independent of the strength of the imposed cooling.

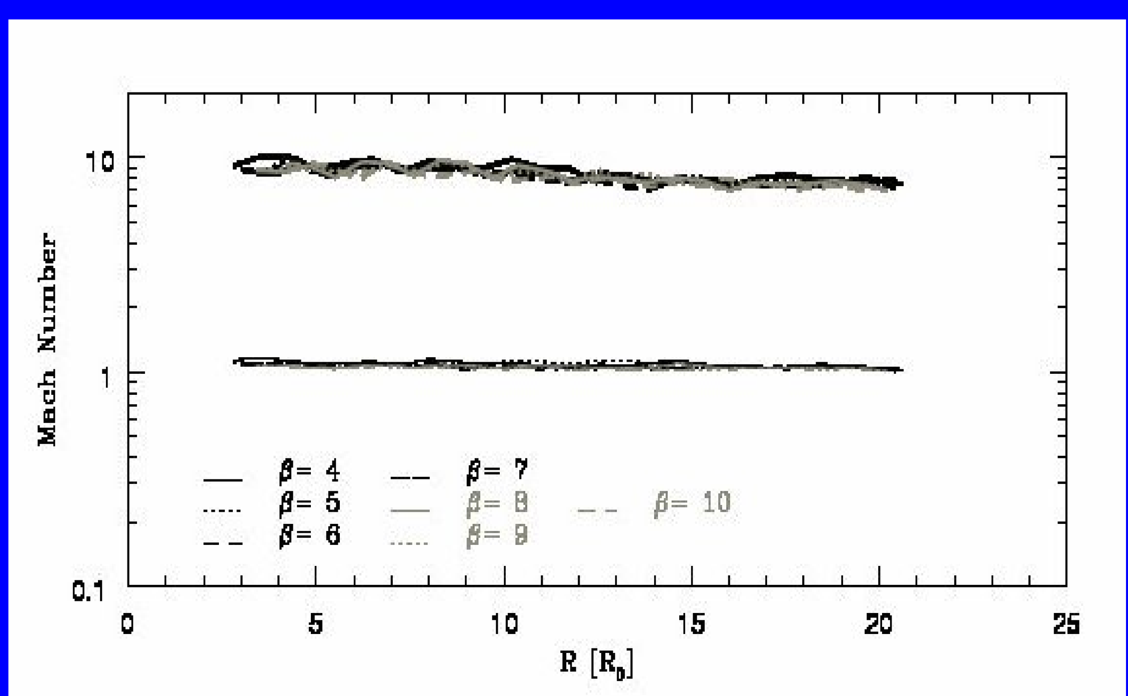

Figure 5: Radial Mach number (thick lines) and Doppler-shifted Mach number (thin lines) as a function of both β and radius

Conclusions

We have determined the relation between the thermal state of the disc and the saturation amplitude of the density perturbations induced by gravitational instabilities. These can easily be determined from the ratio of the local cooling and dynamical timescales. As proto-stellar discs are likely to undergo a period of evolution in the self-gravitating state, this relationship indicates that the more efficient the cooling the greater the density enhancements in the spiral arms, and in turn the higher the rate of planetesimal formation through collisions. Full details can be found in Cossins et al. (2008)

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